

## CONSTRAINED FAIRNESS IN DISTRIBUTION

*Daniel M. Hausman*

GERARD VONG addresses intriguing problems in which it may be impossible to give an equal chance of receiving a good to a set of equal claimants.<sup>1</sup> After developing Vong's views in sections 1 and 2, in section 3, I point out an implausible feature of algorithms that attempt to integrate concerns about comparative fairness and what Vong calls "absolute fairness." I then argue in section 4 against attempting to integrate concerns about comparative and absolute fairness.

### 1. INTRODUCTION

Following John Broome, Vong takes an individual  $Q$  to have a "claim" to a good  $G$  on some agent  $A$  if and only if  $A$  has a *pro tanto* duty to provide  $Q$  with  $G$ .<sup>2</sup> When individuals have equal claims to some good, it seems comparatively fair to give them equal shares of the good or, if the good is indivisible, equal chances of getting the good. In the cases Vong has identified, it is impossible to provide  $G$  to some individuals without also providing it to everyone in some group to which they belong. The good goes to all and only group members. These division problems resemble those discussed by John Taurek, where a drug can save the life of one person or five persons.<sup>3</sup> In these cases, unlike the conflicting claims to some indivisible good that can be possessed by only one person, the distribution of the good determines how many as well as which people get the good, and, *contra* Broome, Vong maintains that equal claimants need not be given equal chances. Indeed, in the case of overlapping groups (where individuals can be benefitted through their membership in more than one group), equal chances may be impossible. For example, suppose that the chance that any of the six individuals  $A, B, C, D, E,$  and  $F$  gets a good  $G$  depends on the chances that

1 Vong, "Weighing Up Weighted Lotteries."

2 Broome, "Fairness"; Vong, "Weighing Up Weighted Lotteries."

3 Taurek, "Should the Numbers Count?"

$G$  will go to one of the following four couples:  $A \& B$ ,  $A \& C$ ,  $D \& E$ , or  $D \& F$ .<sup>4</sup> There is no way to give the six individuals equal, nonzero chances of enjoying  $G$ . All possible lotteries, other than one that gives no one any chance, assign unequal chances to individuals. In this case, Vong suggests that it is fair to give equal chances to each couple, even though that means that individuals  $A$  and  $D$  are twice as likely to receive  $G$  as are the others. What principles imply that this unequal lottery is fairer than others?<sup>5</sup>

Vong maintains that fairness is an amalgam of two species.<sup>6</sup> One is comparative, which counts distributions as fair if chances or shares of the good are in proportion to the strength of claims.<sup>7</sup> The other measures the fairness of a distribution by how many claims it satisfies and by how fully it satisfies them, regardless of comparisons to how fully the claims of other individuals are satisfied. A distribution that awarded everyone half of what they claim, when their claims could have been completely satisfied, is comparatively fair and absolutely unfair.

Vong seeks some criterion that reflects the moral importance of both comparative and absolute fairness.<sup>8</sup> I argue in section 4 that it is better to offer separate assessments of the absolute and comparative fairness of distributions, whose weights vary with context. Until then, I will follow Vong and consider which distributions are fairest “overall.” I shall impose the constraint that lotteries be *efficient*: the probabilities they assign to overlapping groups add up to one and the shares of divisible goods that are assigned to overlapping groups exhaust the good. This constraint can be defended both on the grounds of absolute fairness and on welfarist grounds.

## 2. EXCLUSIVE COMPOSITION-SENSITIVE LOTTERIES

Vong considers several ways to distribute chances among groups in order to treat claimants fairly, and he favors what he calls “exclusive composition-sensitive lotteries” (hereafter EXCS lotteries).<sup>9</sup> The characterization is complicated, and the reader may want to skip to the example in the following paragraph. In EXCS lotteries, each of the  $n$  equal claimants is assigned an initial baseline weight of  $1/n$ .

4 Vong, “Weighing Up Weighted Lotteries,” 324.

5 One answer: it maximizes the minimum chance that any individual will win.

6 Vong, “Weighing Up Weighted Lotteries,” 326–27.

7 Broome, “Fairness.” Like Broome, I regard fairness as comparative, but in this essay I follow Vong’s terminology, expressing later some skepticism about whether absolute and comparative fairness have the same normative source.

8 Vong, “Weighing Up Weighted Lotteries,” 332.

9 Vong, “Weighing Up Weighted Lotteries,” 335.

Each individual  $j$ 's baseline weight is distributed among the groups in which  $j$  is a member. The fraction of  $j$ 's weight assigned to a group depends on how many members in the group are "distributively relevant" to  $j$ , divided by the total number of members distributively relevant to  $j$  in all the groups.<sup>10</sup> A member  $k$  of a group containing  $j$  is distributively relevant to  $j$  in that group if it matters to  $j$  how  $k$ 's baseline probability is distributed among groups. If  $k$  is in some groups that do not include  $j$ , then it matters to  $j$  how  $k$ 's baseline probability is distributed and  $k$  is distributively relevant to  $j$ . If every group containing  $k$  also contains  $j$ , then  $k$  is not distributively relevant to  $j$ . If an individual,  $j$ , is in only one group, then  $j$ 's entirely baseline probability is assigned to that group.

For example, consider:

*Problem\**: There are four equal claimants, Ann, Bill, Chuck, and Diane ( $A$ ,  $B$ ,  $C$ , and  $D$ ). It is possible to distribute chances of getting some good to them only by distributing chances of getting the good to the groups  $A$  &  $B$ ,  $A$  &  $B$  &  $C$ ,  $C$  &  $D$ , and  $B$  &  $C$ . The baseline probability for each individual is  $\frac{1}{4}$ .  $A$  is not distributively relevant to  $B$ , because every group containing  $A$  also contains  $B$ .  $B$  is distributively relevant to  $A$ .

Table 1 lists the distributive relevancies and calculates the chances in the lottery.

Table 1

Group	Distributive Relevancies	Calculation	Chance
$A$ & $B$	$B$ to $A$ (1 of 4)	$\frac{1}{4} \times \frac{1}{4}$	$\frac{1}{16}$
$A$ & $B$ & $C$	$A$ to $C$ (1 of 1); $B$ to $A$ and $C$ (2 of 4); $C$ to $A$ and $B$ (2 of 4)	$\frac{1}{4} (1 + \frac{1}{2} + \frac{1}{2})$	$\frac{1}{2}$
$C$ & $D$	$C$ to $D$ (1 of 4); $D$ 's full baseline (1)	$\frac{1}{4} (1 + \frac{1}{4})$	$\frac{5}{16}$
$B$ & $C$	$B$ to $C$ (1 of 4); $C$ to $B$ (1 of 4)	$\frac{1}{4} (\frac{1}{2})$	$\frac{1}{8}$

We are not quite done. Because it is unfair (and inefficient) to assign any non-zero probability to a subset of another set, the chances assigned to  $A$  &  $B$  and to  $B$  &  $C$  should be distributed among the sets containing these subsets, in this case,  $A$  &  $B$  &  $C$ .<sup>11</sup> In *Problem\**, the EXCS lottery assigns an  $\frac{11}{16}$  chance to  $A$  &  $B$  &  $C$

10 Where I speak of  $k$  being "distributively relevance" to  $j$ , Vong speaks of  $j$  as "exclusive" to  $k$ . I find that this change makes Vong's proposal easier to follow.

11 Vong, "Weighing Up Weighted Lotteries," 342.

and a  $\frac{5}{16}$  chance to  $C \& D$ . This implies:  $\Pr(A) = \Pr(B) = \frac{11}{16}$ ,  $\Pr(D) = \frac{5}{16}$ , and  $\Pr(C) = 1$ .

3. PROBLEMS WITH EXCS AND OTHER LOTTERIES

Because the groups  $B \& C$  and  $A \& B$  are subsets of  $A \& B \& C$ , the EXCS lottery quite rightly gives them no chance of getting the good. Yet, as table 2 shows, the chance that the claims of individuals in different groups are satisfied depends on whether claims could be satisfied via the two subset groups, even though it would never be fair to give them any chance of getting  $G$ .

Table 2

Group	Distributive Relevancies	Calculation	Chance
ABC	C to A and B (2 of 3); A's and B's full baselines (2)	$(\frac{1}{4})(1 + 1 + \frac{2}{3})$	$\frac{2}{3}$
CD	C to D (1 of 3); D's full baseline (1)	$(\frac{1}{4})(1 + \frac{1}{3})$	$\frac{1}{3}$

The lotteries derived in tables 1 and 2 assign chances to the same equal claimants, and both assign nonzero chances only to groups  $A \& B \& C$  and  $C \& D$ . Yet which distribution to the four individuals is fair depends on whether one employs Vong's two-step procedure to decide how to distribute chances among the four groups, or whether one starts by ruling  $A \& B$  and  $B \& C$  out of the lottery on the grounds that they must wind up with a zero probability. In that case, A's and B's chances would be lower ( $\frac{2}{3}$  rather than  $\frac{11}{16}$ ), and D's chances higher ( $\frac{1}{3}$  rather than  $\frac{5}{16}$ ). This result is implausible. Regardless of the status that groups have in other contexts, their only role here is to specify which distributions among individuals are possible. Whether an assignment of chances treats the four equal claimants fairly should not depend on whether they belong to groups to which no chance is given. This is not a bargaining problem, wherein the possibility of individuals getting the good by themselves or via coalitions gives them a threat advantage.<sup>12</sup>

There are alternatives to EXCS lotteries to consider. Suppose one weights each alternative by the proportion of the individual claimants it contains and then multiplies each weight by the reciprocal of the sum of the weights so that the weights add up to one. This method implies that  $\Pr(A) = \Pr(B) = \frac{7}{9}$ ,  $\Pr(D) = \frac{2}{9}$ ,

12 Moreover, since every fair distribution gives C the good, the distribution of C's baseline probability should be irrelevant.

and  $\Pr(C) = 1$ .<sup>13</sup> If, however, one begins by eliminating the groups with zero probabilities, then the chances for the two groups  $A \& B \& C$  and  $C \& D$  should be  $\frac{3}{5}$  and  $\frac{2}{5}$ , and the probabilities among the four individuals are:  $\Pr(A) = \Pr(B) = \frac{3}{5}$ ,  $\Pr(C) = 1$ , and  $\Pr(D) = \frac{2}{5}$ . Proportional lotteries, like EXCS lotteries, imply that the fairest weighted lottery among equal claimants depends on the treatment of groups to which the lottery assigns zero probability.

Vong discusses and criticizes a third method of assigning chances to lotteries, which he calls “equal composition-sensitive lotteries.”<sup>14</sup> In these “EQCS lotteries,” the chance of each group is the sum of fractions consisting of the baseline probabilities for each individual divided by the number of groups in which the individual is a member. The values EQCS lotteries assign to  $A \& B \& C$  and  $C \& D$  also vary depending on how one deals with the zero-probability groups.<sup>15</sup>

There is an easy way to avoid the untoward dependence on membership in groups to which fair lotteries assign no chance: simply delete all groups that are subsets of other groups before calculating the chances. But that solution does not explain why these methods of assigning chances when there are overlapping groups are responsive to whether there are groups to which fair lotteries assign zero probabilities of benefitting. Nor does it help us decide among EXCS, EQCS, and proportional lotteries.<sup>16</sup>

#### 4. ADJUDICATING AMONG LOTTERIES

Vong offers an example that he believes supports employing EXCS lotteries and undermines the employment of EQCS lotteries.<sup>17</sup> I draw different conclusions. Consider the groups,  $G_1$ ,  $G_2$ , and  $G_3$ .  $G_1$  contains claimants 1 through 500.  $G_2$  contains claimants 501 to 1,000.  $G_3$  contains claimants 2 to 999. The EQCS lottery

13 This adopts Frances Kamm’s proportionality proposal (*Morality, Mortality*, 124) and renormalizes so that the weights assigned to groups add up to 1. In this example, the weights assigned to  $A \& B$ ,  $A \& B \& C$ ,  $C \& D$ , and  $BC$  would be  $\frac{2}{4}$ ,  $\frac{3}{4}$ ,  $\frac{2}{4}$ ,  $\frac{2}{4}$ . The sum is  $\frac{9}{4}$ . Multiplying by  $\frac{4}{9}$ , the groups’ chances would be  $\frac{2}{9}$ ,  $\frac{1}{3}$ ,  $\frac{2}{9}$ , and  $\frac{2}{9}$ . Donating  $B \& C$ ’s and  $A \& B$ ’s probabilities to  $A \& B \& C$ , the result is  $\Pr(A \& B \& C) = \frac{7}{9}$  and  $\Pr(C \& D) = \frac{2}{9}$ .

14 Vong, “Weighing Up Weighted Lotteries,” 334.

15 In this example,  $\Pr(A \& B) = \frac{5}{24}$ ,  $\Pr(A \& B \& C) = \frac{7}{24}$ ,  $\Pr(C \& D) = \frac{1}{3}$ , and  $\Pr(B \& C) = \frac{1}{6}$ . The fair lottery if one starts with four groups gives  $A$  and  $B$  a  $\frac{2}{3}$  chance ( $\frac{(5+7+4)}{24}$ ) and  $D$  a  $\frac{1}{3}$  chance. If one starts with two groups,  $A$  and  $B$  each have a  $\frac{5}{8}$  chance while  $D$  has a  $\frac{3}{8}$  chance.  $C$ , of course, is sure to win.

16 Vong (“Weighing Up Weighted Lotteries,” 338) also discusses an iterated version of Timmerman’s individualist lottery, which I shall not discuss; see Timmermann, “The Individualist Lottery.”

17 Vong, “Weighing Up Weighted Lotteries,” 339–40.

assigns a chance of a little more than a quarter to each of  $G_1$  and  $G_2$ , and a little less than one half to  $G_3$ .

Vong finds this result intolerable:

A theory of fairness that utilizes the equal composition-sensitive lottery procedure gives the startlingly implausible result that it is fair to give a greater than 50 percent chance to save [members of] either one of  $G_1$  or  $G_2$ , making it more likely that 500 claimants rather than 998 claimants will be saved. This is an affront to absolute fairness because benefiting the much larger group of 998 claimants is less likely than benefiting one of the much smaller groups containing 500 claimants.<sup>18</sup>

Vong's EXCS lottery, in contrast, gives about a 96 percent chance to  $G_3$ . The EXCS lottery probably satisfies many more claims than the EQCS lottery. It is far fairer absolutely. However, Vong's EXCS lottery gives individuals 1 and 1,000 a vastly lower 2 percent chance of getting  $G$ . On Broome's view of comparative fairness as requiring equal chances for equal claimants,  $G_1$  and  $G_2$  should have equal chances of  $\frac{1}{2}$ . On Kamm's proportional view with the renormalization discussed above,  $G_1$  and  $G_2$  should have a little more than a 25 percent chance and  $G_3$  a little under a 50 percent chance. So individuals 1 and 1,000 will have about a 25 percent chance of getting the good, while everyone else will have about a  $\frac{3}{4}$  chance. This seems fairer comparatively, but, as Vong argues, less fair absolutely. Vong's proposal, with its focus on distributive relevance—that is, whether  $j$ 's benefitting affects  $k$ 's benefitting—makes the magnitude of expected claim satisfaction the dominant factor here: the larger the chance of  $G_3$ , the greater the "absolute" fairness.

There are two moral considerations here—in Vong's terminology, absolute and comparative fairness. Whereas Vong sees these as two faces of the same coin, I see one as a matter of how one shows respect to individuals, while the other is focused on satisfying duties to individuals. What is absolutely fairest is to give the good to  $G_3$ , which fully satisfies 998 claims. What is, on Broome's view, fairest comparatively is to give everyone the same  $\frac{1}{2}$  chance by giving that chance to  $G_1$  and  $G_2$ . Vong accepts the comparative unfairness of the EXCS lottery, because he seeks a rule for assigning chances that integrates absolute and comparative fairness.

I think that Vong's search for a context-invariant compromise between absolute and comparative fairness is a mistake. It is more perspicuous to separate the questions concerning absolute and comparative fairness and to allow the trade-

18 Vong, "Weighing Up Weighted Lotteries," 340.

off to respond to details of the specific circumstances, which may include other ethically relevant aspects. These sometimes call for compromises and sometimes respond to one consideration, passing over the other. In the case concerning  $G_1$ ,  $G_2$ , and  $G_3$ , what is comparatively fairest is so different from what is absolutely fairest that compromises are not plausible: one should give the good to  $G_3$  despite its comparative unfairness if the good that individuals have claims to is a lifesaving medicine. This is far better on the grounds of well-being as well as absolute fairness. On the other hand, if the good were seats at a presidential inauguration, it may be more important that people be treated equally than that so many more with claims to attend are able to do so.

There are other cases where the demands of absolute and comparative fairness should affect the distribution. The quandaries concerning the allocation of COVID-19 vaccines might be examples. What I am questioning is whether integrations of comparative and absolute fairness concerns, such as EXCS, EQCS, and proportional lotteries, are helpful in guiding ethical decisions. Having determined what is comparatively fairest and what is absolutely fairest, one needs to decide how to distribute the chances, taking into account other relevant moral considerations. There is no reason to insist on a uniform adjudication of just two of the considerations.

Depending on the method and whether one ignores the two groups in Problem\* to which no chance is given, we have seen arguments for several different assignments of chances. In the specific problem, all of the different ways of apportioning chances among the four individuals seem plausible in the abstract. I see no good argument for defending one of these as the overall fairest without attending to the characteristics of the good and of the claims to the good.

Eschewing the determination of which distribution is fairest overall leaves one with the tasks of judging which distributions are fairest comparatively and which are fairest absolutely. I suggest that the comparatively fairest distribution assigns shares and chances to equal claimants that are as equal as possible or that maximizes the minimum chance of receiving the good. In the case of Problem\*, giving  $A$  and  $B$  a  $\frac{6}{11}$  chance and  $D$  a  $\frac{5}{11}$  chance minimizes the variance. However, giving  $A$ ,  $B$ , and  $D$  each a one-half chance of getting  $G$  maximizes the minimum chance and perfectly equalizes the chances for everyone except  $C$ , who is in any case guaranteed to get the good and whose chance is hence arguably irrelevant to which distribution is fairest. Absolute fairness is not simple either, unless it is just a matter of how many claims are satisfied, as is the case here, where giving the good to the group  $A \& B \& C$  guarantees that three of the four individuals will have their claims satisfied. Which distribution is overall fairest, let alone best, all things considered, depends on the context. It may be what is comparatively fair-

est, what is absolutely fairest, some compromise, or an unfair distribution that is ethically attractive on other grounds.

#### 5. CONCLUSION

Overlapping groups pose theoretical problems concerning how to distribute goods or chances fairly. Compromises such as Vong's EXCS lotteries have implausible implications, which can be avoided by addressing separately the comparative and absolute fairness of distributions of chances or goods. Rather than seeking some general algorithm to assign the proper significance to these separate moral considerations, allocators should look to the details of the context to prioritize these separate considerations of fairness and other relevant ethical considerations such as well-being.<sup>19</sup>

Rutgers University  
dhausman@cplb.rutgers.edu

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